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A mathematical toy model of R&D process. How this model may be useful in studying territorial development

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ABSTRACT

This work describes a mathematical application of a technological model of the R&D process, presented in a previous work, with the objective to contribute to a better knowledge of relation between R&D investments and growth. The model considers R&D as an organized flux of knowledge and capitals generating new technologies and a general knowledge exploitable for further R&D activities. The mathematical model makes an oversimplification of the R&D activity considering R&D investments related to number of R&D projects carried out, and economic growth, stagnation or decline, related to the number of new technologies entering in use. The model considers the circulating knowledge in a territory in term of number of information packages generated by R&D projects and external contributions in term of scientific, technical or other knowledge. A combinatory process with all available packages gives the total number of potential innovative ideas, part of them generating R&D project proposals. The ratio between the number of R&D proposals and the total number of potential innovative ideas may be considered related to the *innovative system efficiency* of the territory. Proposals are selected forming the number of R&D projects effectively carried out following the adopted strategies for financing R&D projects. The number of new technologies entering in use depends on a selection rate of all R&D projects carried out, and the number of new successful technologies with high rates of return of investment depends on a selection rate of all new technologies entering in use. The study considers an application of the model consisting in the introduction of a variable number of initial R&D projects in a territory with various degrees of innovative efficiency resulting or not, after a certain time, in entering in use of new technologies and possible successful technologies. Calculations show that dependence curves, in term of number of carried out R&D projects as a function of the innovative efficiency of the territory, and following dependence of formation or not of new or successful technologies, delimit three specific areas in the diagram corresponding to development, stagnation and decline of the technological asset of the territory. The results of calculations of the model show how complex is the relation between R&D investments and economic growth, characterized by absence or weak growth at level of R&D investments under a critical value, and exponential growth above due to the autocatalytic effect of R&D. This discontinuity resulting by the model calculations is in contrast with assumed continuity of dependence of growth by R&D investments often considered in econometric models.

KEYWORDS

Technology innovation, Research & development, R&D model, R&D management, Socio-economic growth, Territorial development

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1 Introduction

There is an important amount of studies concerning the relation between investments in R&D and resulting economic growth with the aim to give to governments and local authorities indications about promotion of economic growth through R&D investments. This relation has been object recently of an empirical survey (Becker 2013) in which it has been observed that the dependence of R&D determinants on economic growth assumes graphically often an inverted U shape. Such type of dependence, as a quadratic curve, has been found for example in an interesting study on dependence of gross domestic product per capita (GDP) as a function of gross domestic expenditure on R&D, expressed as percentage of GDP for numerous countries (GERD), calculated as average in the period 1996-2001 (Coccia 2008). In this study industrialized countries such as France, Germany, United Kingdom and Sweden have shown a good agreement with the curve, but not in the case of Italy, USA and Japan, the first two with GDP values higher and the last lower than the curve. Actually, our model of the R&D process used for this study does not show any decrease of generation of new technologies, and indirectly of growth, with the increase of R&D investments contrarily to the observed inversed U behaviour in various empirical studies. Following what it has been already discussed in the previous work (Bonomi 2017), such decrease of GDP at higher values of GERD does not have in fact any satisfactory explanation. It may be also observed that in the last decades the values of GERD of many industrialized countries show only limited variations. This fact poses the question whether dependence of GDP on GERD may represent a dynamics of GDP evolution with GERD or not. In fact, the Lisbon strategy for EU countries considers that a sensible increase of GERD to a maximum value around 3% would result in a consequent sensible increase of GDP, however this suggestion is not supported by any dynamics of statistical data (Bonomi 2017). We may argue that the observed dependence of GDP by GERD might be actually representing simply the nearly static situation of the different innovative systems of the various countries (Bonomi 2017). That could explain the observed deviations for Italy and USA from the quadratic curve that, in the case of Italy, might be due to the existence of unregistered R&D investments by SMEs (Hall, Lotti, Mairesse 2009), and in the case of USA, to a better exploitation of scientific results for innovations (Ben-David 1968), showing in this way the importance of the specific innovative system of a country in the relations between GERD and GDP. Concluding, it may be suggested that economic growth in industrialized countries should not depend actually by R&D investments, that might be considered rather a means, but by the intensity of generation of innovative ideas, that depends on the efficiency of the innovative system, and strategies and availability of capitals financing their development joined with an effective industrial organization (Bonomi 2017).

The aim of this study is to give a contribution to a better view on the complex relation between R&D investments and economic growth in a territory by a quantitative approach simulating mathematically this relation through the model of the R&D process previously developed (Bonomi 2017). The basic idea is that *investments in R&D may be simulated by the number of R&D projects carried out in a territory, and that economic growth may be simulated by the number of successful new technologies entered in use*. We consider then in this work an attempt to apply the model quantitatively to the problem of economic growth of territories, in particularly characterized by low or declining industrial activities, by investing in R&D projects. The quantitative simulation studies the effect of starting a certain number of R&D projects in a territory and the possibility to generate a certain number of new successful technologies triggering a socio-economic growth. The objective is to get an idea about conditions in which such intervention might be a success. R&D is a quite complex process and, in our attempt, we have considered in fact only a simulation of such process, furthermore, many parameters, necessary to operate quantitatively the model, are in fact not really available and should be substituted by reasonable but only indicative values suggested by experience in R&D activity. For this reason, we have preferred to consider our mathematical model as in fact a *toy model* that, however, we think it could show some interesting aspects of the dependence of growth on R&D investments.

In the model, R&D is considered as an organizing activity of a looping double flux of knowledge and capitals, generating new technologies and an R&D general knowledge (GRDK). In particular, such general knowledge, generated either by R&D projects leading to new technologies or abandoned developments, has been considered the driving force for new R&D projects. In Fig. 1 we have reported a schematic view of the double flux model of R&D. In this model, the input for the R&D process is constituted by R&D projects determined by R&D investments financing R&D project proposals. The R&D process is seen as an activity based on projects, and has as output new technologies and GRDK that, combined with scientific, technical and other external knowledge, generates new innovative ideas and consequent new proposals for R&D projects. The use of new technologies will require capitals and be a source of possible return of investments. On the other side, the socio-economic system determines new private or public investments for R&D constituting the total R&D investment available in a territory. In this way, the right side of Fig. 1 consists in a looping flux of knowledge constituted by GRDK, external contributions and elaborated projects proposals. The left side of Fig. 1 represents a flux of capitals constituted by development costs of new technologies seen as investments, industrial capitals for the use of technologies, return of investments and a flux of financing for R&D activity in the territory.

After this introduction in a second section, we present a description of mathematical aspects of the model allowing calculation of number of generated new and possibly successful technologies in a territory. In a third section, we present an estimation of the various parameters that may be used to run the model and in a fourth section the results of our calculations. These results are discussed in a fifth section considering the influence of the various parameters on generation of successful technologies in a territory, and in the sixth section, we present some general conclusions of this study.

2 DESCRIPTION OF THE MATHEMATCAL MODEL OF R&D

The calculations with mathematical R&D model have the aim to obtain results about the economic growth in a territory by starting a certain number of R&D projects and development of endogenous new technologies able to have a positive socio-economic impact. For this purpose, we study the effect of a variable number of initial R&D projects on possible generation of new technologies, and exploitation of GRDK and external knowledge, as a function of efficiency of the territory in the generation of a further number of R&D projects proposals. The knowledge available in a territory is considered formed by a certain number of information packages. We assume in these calculations that there are always enough R&D investments to cover budget of all valid proposals generated by the innovation system of the territory. We suppose also that the technological innovative system of the territory operates by a more or less advanced regime of open innovation (Chesbrough 2003), and that technology innovation is carried out following a more or less advanced distributed innovation system (Haour 2004). That means that actors carrying out R&D projects are not limited to industrial R&D laboratories but also by contract research laboratories, private or public research laboratories, start up, etc. exchanging in a certain measure the generated GRDK in the territory (Bonomi 2017). The R&D activity is seen in term of projects and, although normally a new technology is generated by a sequence of R&D projects, for simplification in our application we consider that a new technology may be generated by a single project. Actually, generation of R&D project proposals, starting of new R&D projects and formation of new technologies are a continually in action, however, in our application, for simplification of computation, we consider the R&D activity in term of cycles, each fed by a certain number of R&D projects, and generating or not new technologies. That means that duration of generation of R&D project proposals followed by carrying out of R&D projects is the same for all projects and equal to the cycle time. As knowledge may be partly lost with time by a fading effect, we have assumed a percentage of loss of knowledge, in term of number of information packages, occurring at each cycle and concerning all remaining knowledge of previous cycles. Concerning the socio-economic growth of the territory, we have considered, in a simplified view, that it is proportional to the resulting number of new successful technologies that are in fact a part of all new technologies entering in use.

For the definition of parameters of the model, we may separate those concerning the flux of capitals from those concerning the flux of knowledge. In the case of flux of capitals, instead of return of investments, we have considered simply the generated number of new and possibly successful technologies using two rates in term of percentage of R&D projects that generates new technologies, and percentage of new technologies that are successful. Considering N the number of R&D projects carried out in a cycle, the number T of new technologies entering in use will be determined by a selection rate v following the formula:

$$T = vN \quad (1)$$

Considering now the successful number of new technologies S, they will be the result of a selection rate r on the number T of new technologies entering in use following the formula:

$$S = rT \quad (2)$$

Concerning the flux of knowledge, we have, first of all, to define a measure of knowledge generated by R&D projects in term of number of information packages.

For this purpose we consider that each R&D project generates an average number p of information packages and that total available information packages result of the sum of packages generated by the cycle plus the information packages of previous cycles reduced by the fading rate effect f. Such total number of packages shall be increased by a contribution taking account of information packages coming from scientific, technical and other information composing an external available knowledge estimated as a fraction E of available information from projects.

The total number of information packages coming from carried out R&D projects, reduced by fading effect on number of packages generated in past cycles, and added with number of external information packages of scientific or technical nature, represents all the available packages for the generation of innovative ideas, and then R&D project proposals.

This number, indicated as I_T, may be calculated mathematically by the formula:

$$I_T = \left(N_L p + \sum_{i=1}^{n} I_i(1 - f) \right)(1 + E) \quad (3)$$

in which we have:

I_T : total number of information packages available for new innovative ideas after the last cycle

N_L : number of R&D projects in the last considered cycle

p : average number of information packages of each R&D project
n : number of past cycles
I_i : number of remaining information packages of past cycles from i = 1 to i = n)
f : rate of fading effect (*)
E = fraction of added information packages by external knowledge

(*) It shall be noted that for remaining information packages of past cycles we intend that initial information packages of a cycle is reduced by fading effect *f* at each successive cycle before the last one. With $f = 0$ the fading effect is not present and with $f = 1$ there is a complete loss of past information packages.

The number of potential innovative ideas G, independently of their validity, is obtained by a combinatory calculation considering the number of available information packages I_T, and number of combining information packages *m* necessary to have a potential innovative idea, and expressed by the following formula:

$$G = I_T(I_T - 1)/m \quad (4)$$

in which we have:

G : number of potential ideas for innovations
I_T : total number of information packages available for potential innovative ideas after the last cycle
m : combinatory number of information packages necessary to generate a potential innovative idea

In fact, such number G of potential ideas is a simple combinatory result not considering any validity about specific combinations and contains necessarily in fact a large number of invalid or even absurd combinations. It is the task of the territorial innovative system to make a selection of valid innovative ideas. The number P of effective new valid ideas becoming R&D research proposals may be obtained by considering a rate factor *s* applied to the number G of potential new innovative ideas:

$$P = sG \quad (5)$$

Such rate *s*, expressed as percentage, represents a measure of the *innovative system efficiency* (ISE) of a territory. A last selection occurs in comparing R&D project proposals budgets with available R&D investments, and we may define a rate *t* determining the number N of R&D proposals that can effectively become R&D projects following the relation:

$$N = tP \quad (6)$$

Concluding we may express the total number N of R&D projects carried out in a cycle as a function of generated packages of information I_T by the previous cycles combining equations (4), (5) and (6):

$$N = tsI_T(I_T - 1)/m \quad (7)$$

However, in simplifying our application, we will consider later that all generated R&D proposals are valid and there is always enough R&D investment for the corresponding R&D projects. That means that we consider always $t = 1$ and consequently the number N of R&D projects will be equal to the number P of generated R&D proposals.

Considering the adopted functioning of the model with its simplifications we may expect the formation of three scenarios. The first one corresponds to the case of introduction of a limited number of initial R&D projects and a low ISE, by consequence the number of R&D projects will be insufficient statistically to generate a new technology entering in use. On the other side a low efficiency in exploiting GRDK may lead to a number of new proposals inferior to the initial number of R&D projects decreasing the number of projects with the number of cycles that, if not enough compensated by available past knowledge, will lead to a situation of decline and abandon of the R&D activity by lack of innovative ideas. In the second one there is a sufficient number of initial R&D projects and acceptable ISE with an increase of the available GRDK, however the number of generated new technologies, after a reasonable number of cycles, may be statistically insufficient to have at least one successful technology to implement the socio-economic growth of the territory. In this case, the technology evolves but without assuring a real development eventually entering in a stagnation phase typically of the Red Queen regime (Bonomi, Marchisio 2016). In the third one, there is a sufficient number of initial R&D projects and a good innovative system originating a reasonable high number of new technologies and possibly successful technologies achieving a socio-economic growth of the territory. In this case, GRDK increases rapidly with the sequence of cycles and then the number of financed R&D projects becomes potentially enormous although in the reality it will be limited by actual availability of R&D investments or by available human resources and structures for R&D activity of the territory.

3 PARAMETERS OF THE MODEL

In order to make calculations it is necessary to establish the value of the various parameters used for calculations with the model. As cited previously, most of statistical data necessary for parameters are practically not available and we might consider only reasonable indicative estimations made possible by experience in R&D activity. The only parameter that might have a relation with statistical studies concerns the rate *r* determining the success of a technology following equation (2). In fact, we have a study about value of patents showing their skew distribution (Scherer, Haroff 2000). This study considers various groups of patents the greatest including 772 German patents hold valid for at least ten years. An elaboration of data in this case shows that less than 1% of patents have very high outcomes and that only about 20% of patents result in substantial outcomes (Bonomi, Marchisio 2016). Considering these results we may indicate a value of 0.2 for the parameter *r* selecting the number of successful new technologies. Much more difficult is the estimation of parameter *v* determining the number of new technologies entering in use in respect to the number N of R&D projects carried out in a cycle following equation (1). The rate of success of R&D projects becoming a new technology is quite variable depending, beside socio-economic factors, on the radical degree of the new technology, while rate of discontinuation of R&D projects is different depending on the reached phase of the innovation process (Bonomi 2017). It is well known for example that the development phase of the innovation process, in respect to the feasibility phase, is particularly selective and it is also called the “Valley of Death” of the projects (Auerswald, Branscombe 2003). On the other side an R&D project concerning an innovation with a limited novelty or low radical degree may have a much more chance to become a used technology. Taking account of previous considerations and experience in R&D we might indicate a number of 40 R&D projects necessary to obtain a new technology entering in use and then an indicative value for parameter *v* of 0.025 that however should be considered the most uncertain parameter value of the model. The assigned parameters of *r* and *v* are set as constant in the studied application of the model. That means that on average it is necessary a total number of 200 R&D projects to generate five new technologies entering in use and only one technology of these five be successful triggering a sensible positive socio-economic impact on the territory. Considering now parameters concerning the flux of knowledge of the R&D model, we have already defined the measure of knowledge in term of number of information packages circulating in the flux. Quantitative data on generation of information packages by R&D projects and number of packages necessary for the combinatory calculation of innovative ideas are not available but experience in R&D indicates that they cannot be for a single project a very high number.

Table 1. Parameters used for application of the model and their values and ranges

Parameters	Value /range
r rate of success of technologies	0.2
v rate of generation of new technologies	0.025
t rate or R&D proposals selected for R&D projects	1
p number of information packages per R&D project	3
m combinatory number of information packages	2
f rate of fading effect	0.5
E rate of external information contribute	0.1
N_o number of initial R&D projects (range)	10 -100
s rate of selection of innovative combinatory ideas for proposals (range)	0.001 – 0.01

For this reason we have considered, in a conservative view, a number of three information packages, associated to each project, for parameter *p*, and for parameter *m* an average number of two for information packages necessary to the generation of innovative ideas. Another parameter necessary for calculation concerns the fading effect on information packages generated in past cycles and including past external information. It has been considered that about 50% of past information is lost at each cycle. That means the total number of past information packages is halved at each cycle. The fading effect is then established to a value of 0.5 for the parameter *f*. The external contribution of information packages coming from scientific, technical or other types of information to the total information packages available for generation of R&D projects cannot be very high in respect to GRDK. For this reason, we suggest for the external contribution an indicative added value of 10% of the total information packages generated by GRDK establishing a value of 0.1 for parameter *E*. Finally, there are two variables that are used for the parametric study of the model application that concern the initial number N_o of R&D projects and the rate of selection of innovative ideas becoming R&D proposals and then R&D projects. Concerning the initial number N_o, we have considered a range from 10 to 100 projects. Considering the rate of selection *s* of potential combinatory number of ideas G, we have established a range between 0.001 – 0.01, corresponding to values of ISE of 0.1% - 1%, to calculate the number of combinatory ideas valid for R&D proposals. In Table 1 we have summarized the values and ranges of the various parameters used for application of the model.

The variable parameters calculated in the application of the model will be:

I_T: total number of information packages available for starting a cycle
G: total number of potential combinatory ideas
P: total number of R&D project proposals
N: total number of R&D projects carried out in a cycle
T: number of new technologies entering in use
S: number of new successful technologies

Finally, we have considered a maximum number of cycles characterizing the effects of introduction of initial R&D projects N_o. Such number has been established to 5 cycles.

4 RESULTS OF THE MODEL APPLICATIONS

Before entering in presentation of results of calculations with the model, we would stress the fact that this model is not a real reproduction of the complex activity of R&D, but only a simulation in which quantitative results are only indicative depending on choice of parameters values that in fact are not resulting from any real statistical data but only from reasonable values suggested by experience. However, although the adopted simplification, the model may give an idea on generation or selection processes occurring in a real R&D activity in accord with experience. Model calculations have been simply implemented using an EXCEL® sheet. The calculation starts by introducing an initial number of R&D projects for the first cycle and calculating the number of information packages formed by R&D. This number, increased with external information, gives the total number of information packages available for generation of R&D project proposals whose number will depend on the adopted ISE expressed by the rate *s*. As already established we consider in this application a full availability of R&D investments for proposals and a number of R&D projects equal to the number of proposals. Should the number of R&D projects be enough high, there will be generation of new technologies following the respective adopted rate *v*, and possibly successful technologies following the respective adopted rate *r*. The number of R&D projects generates a further number of information packages, added to past-generated packages reduced by fading effect, and added to external information to give a new number of total information packages allowing starting of calculation for the second cycle. A total of five cycles have been used to evaluate the effect of the initial number of R&D projects and the adopted ISE expressed as rate *s*. An example of the model as appearing in an EXCEL® sheet is reported in Fig. 2, and it represents calculations results at the fifth cycle, using an initial number of R&D projects N_o equal to 24, and obtaining a single successful technology in an innovation system with an efficiency value ISE corresponding to s = 0.005 (0.5%).

In a first run of calculations we have determined the minimum number of initial R&D projects that are able to generate a number of R&D projects higher than the initial one following the adopted ISE value. Obtained results are reported in Fig. 3. We may observe that in the case of the lowest ISE value equal to 0.01%, the number of initial R&D projects is very high (almost 200 projects) to obtain the expected result. Such number may be considered excessive and unrealistic for a weak innovative system. In fact, using a low initial number of projects, it will result in this case a continuous decrease of generation of new R&D projects eventually terminating the R&D activity. For this reason, we have considered for further calculations only an ISE value of at least 0.25% to a maximum of 1%. Looking to Fig. 3 the curve separates in fact the diagram area in two zones. Below the curve, the points represent a situation of technology stagnation and possibly decline, while above the curve the points represent a situation of technology development but not necessarily of socio-economic growth if none of new technologies becomes a successful technology. It should be noted that, in certain conditions, although existing an initial lower number of generated R&D projects, such number may become higher after a certain number cycles because of cumulated past information although limited by the fading effect. In a second run of calculations we have determined the minimum number of initial R&D projects necessary to obtain at least one successful technology within a maximum of 5 cycles as a function of ISE values. Results are reported on Fig. 4. Also in this case the curve separates two zones, the area above the curve representing a space with points of technology growth, as corresponding to generation of successful new technologies. The area below the curve represents points of technology decline or stagnation by simple formation of new but not successful technologies. Using data obtained in this second run we may determine the minimum number of projects resulting by the sum of projects of the various cycles that gives the generation respectively of at least one new technology and at least one successful technology as a function of adopted ISE. By model calculations a new technology, resulting by a minimum of initial projects, is obtained at the fourth cycle and a successful technology in the following fifth cycle. The obtained two curves are reported on Fig. 5. In this figure, there are three areas: the first one, above the higher curve, represents a zone of points corresponding to generation of successful technologies and then of growth, the second one below the lower curve represents a zone with points corresponding to absence of generation of new technologies and then of decline, the third one between the two curves represents a zone with points corresponding to generation of new but not successful technologies without a real influence on growth. This last case corresponds to presence of new technologies but not of competitive technological advantages characterizing successful technologies and that is typical of a Red Queen regime (Bonomi, Marchisio 2016). Finally, in Fig. 6, we have reported results of a third run about the evolution of the number of new and successful technologies as a function of number of successive cycles. For this calculation we have chosen an initial number of R&D projects equal to 50, and generation of new technologies and successful technologies using two intermediate value of ISE expressed by rates s=0.005 and s=0.0025 corresponding respectively to a high and a low efficiency. Reported results show easily that for high efficiency (s=0.005) there is an exponential growth of number of new technologies, and then of successful technologies, in accord to what it is expected by the R&D model when financing of R&D projects is not limited. For the high value of ISE, at the fourth cycle the calculated number of new technologies is of 2828 and successful technologies of 566, not represented as out of scale. Calculation made with a low ISE (s=0.0025) there is, considering six cycles, only a limited increase of new technologies and a small linear increase of successful technologies. Of course, the extremely high growth of technologies in a territory, observed for high ISE, is not realistic, and limitations of number of financed R&D projects could appear facing such high number of potential R&D projects, and another limitation may be simply constituted by lack of structures or human resources to carry out such huge number of R&D projects.

5 INFLUENCE OF PARAMETERS ON MODEL RESULTS

The influence of parameters on model results depends on how the model operates and, although it is only a rough simulation of R&D activity, it might suggest interesting observations about the real functioning of the R&D process. In fact, the model operates through a sequence of generative and selective process steps. Generative steps concern the combinatory process among information packages, constituted by general R&D knowledge GRDK, and external information, while the number of available R&D project proposals is determined by the innovation system efficiency ISE of the territory. Selective parameters determine the number of financed R&D proposals, the number of R&D projects generating new technologies, and the number of successful technologies appearing among the generated new technologies. All selective parameters induce a linear dependence between number of projects proposals and resulting number of technologies, while the generation process of innovative ideas induces an exponential dependence because of the combinatory process at the base of calculations. In fact, the presence of selective parameters makes the existence of a critical minimum number of projects and only above this number it will be possible the statistical generation of new or successful technologies. On the other side, the combinatory process makes the generation of R&D projects, and then new or successful technologies, an autocatalytic process of increasing growth. Experience in R&D activities demonstrates the validity of such results. In fact, it is well known that low investments in R&D give poor values of growth, while effective territorial innovation systems, joined with full availability of R&D investments, are able to make an exponential growth of technologies and positive socio-economic impacts, as observed for example in the case of Silicon Valley (Bonomi 2017). After these general aspects of parameters influence, it is also interesting to examine in detail the effects of specific parameters of the model related to the real situation of the R&D activity:

Generation of innovative ideas

The model of R&D activity (Bonomi 2017), and the present mathematical version, considers as main source of innovative ideas the GRDK formed in R&D activity joined with external information mainly of scientific and technical nature. The mathematical model treats such knowledge as an ensemble of information packages and new ideas are formed by a combinatory process of such packages. The origin of new technologies, by exploitation of scientific information with combination of pre-existent technologies, has been already considered discussing the nature of technology (Arthur 2009), and new technologies may be formed also by simple combinatory processes without recurring to specific scientific discoveries of new phenomena (Bonomi, Marchisio 2016). Such combinatory process is expressed mathematically by the model in equation 4. However, although information packages cumulate with time (number of cycles following the model), it is necessary to consider also a loss with time of such information calculable mathematically using a fading parameter. In equation 3, the model establishes the way it is treated the fading effect for calculation of the total information packages available for generation of innovative ideas. It should be noted that fading effects may be different following the type of structure in which the R&D activity is carried out. Industrial R&D laboratories may have higher fading effects because part of exploitable GRDK available for new technology developments could not be of interest in the frame of strategies of a firm and lost. On the contrary, in contract research laboratories, as well as in public or private research laboratories the fading effect is lower as all generated GRDK may be potentially used for new research projects independently of type of developed technology. Existence of fading effects has been shown in a previous work about exploitation of GRDK describing some real examples (Bonomi 2017). That brings to the conclusion that R&D activities external to industry may give a better contribute in term of accumulation of useful information for generation of innovative ideas and are important for the territorial innovative system.

Generation of R&D project proposals

The number of combinatory possibilities easily reaches great figures considering that its growth is exponential following equation 3 and 4 used in the model. However, most of possible combinations are irrelevant or even absurd and it is the task of the innovative system to find the good combinations for valid R&D project proposals. The mathematical model for this purpose uses simply the rate coefficient *s* corresponding in percentage to the innovative system efficiency ISE to calculate the number of R&D project proposals from the total number of potential combinations. Actually, ISE is the result of a complex interaction of various factors. An important one concerns the individual creativity of researchers that it is encouraged by favorable organization attitudes and self determination and assessment (Dumbleton 1986), but new innovative ideas may result also by generative relations during discussion among various partners interested in R&D with the emergence of new proposals for potential new technologies (Lane, Maxfield 1995). Another aspect concerns the entrepreneurial attitude of researchers versus scientific research results, or available GRDK, with the figure of researcher that may be considered also as an entrepreneur (Boehm, Groner 1972). This entrepreneurial view of research constitutes in a certain way a gap between USA and Europe, this last considering historically scientific research more as a cultural activity and giving a minor interest to applications (Ben-David 1968), also confirmed by more recent studies in UK (Lam 2011) and in Italy (Bonomi 2014), and justifying in a certain way the historical USA domination in R&D.

Selection of R&D proposals for financing

Selection of R&D proposals for R&D projects financing is the result of various factors including strategies, availability of capitals, validity of proposals, etc. that however the model cannot take into account. In fact, calculations made in this work consider, for simplicity, that all proposals are financed as R&D projects. In fact, there are three types of possible scenarios already discussed in a previous work on the R&D model (Bonomi 2017). The most diffused case is the limitation of number of financed projects dues to availability of investment capitals or public aids. Another possible case is a limitation due to the available number of proposals in respect to availability of investments, generally in form of public aids, happening in particular in low developed or industrial declining territories. Finally, a less frequent situation is the full availability of capitals for investments in all valid R&D research proposals as observed in the Silicon Valley. Interesting in this case is the existing difference in financing strategies of venture capital in Europe and USA especially in the case of Silicon Valley as described in a previous report (Bonomi 2016). In the case of Europe selection coefficient *t* is low determining a limited number of accepted proposals and selection is made mostly on the base of an estimation of possible success of the projects. In the case of the Silicon Valley selection coefficient *t* is high allowing financing of a greater number of proposals, and selection made mostly on the base of possible high returns of investments of the projects. The result is that, in the case of startup, the percent of success, expressed indicatively by the model with coefficient *r* of generation of successful technologies, is about 20 -25% in Europe but only about 5% in the Silicon Valley. However, positive exits of start up in the Silicon Valley have a much higher return of investments making a much higher economical impact than in Europe (Bonomi 2017). In a certain way, it is possible to affirm that European strategies tends to increase the value of parameter *v* for generation of new technologies by a hard selection of R&D proposals, while American strategy tends to increase the value of parameter *r* for generation of successful technologies by looking prevalently to their potential return of investment considering estimation of technical feasibility too much uncertain to be useful for a selection. Considering the various firms strategies in financing R&D there are three cases. The first one concerns the absence of R&D investments. In this case possible economical losses by R&D investments are of course absent, however, on the medium or long term, the technological competitiveness of firms may decrease putting in danger their survival. The second case concerns firms or territories that invest in R&D with an amount under the critical value with absence of generation of successful technologies and return of investments of possible new technologies that does not cover the amount of R&D investments. In the third case, the investments in R&D are above the critical values and we may observe an autocatalytic economic growth for the firms. Paradoxically, the first case is economically more valid than the second one as it does not have any loss due to R&D investments although both cases will have in the future a loss of technological competitiveness. A situation close to the first case has been observed for example in the history of development of Italian industrial districts in which innovations, of simple combinatory type, were made in workshop, without recurring to any R&D activity, have however lead to their development (Hall, Lotti, Mairesse 2009). However, it is not certain whether such development could be maintained in the future. In the second case, there is a conservative attitude that limits R&D investments because of fear of unsuccessful exits, leading to a poor or even absent generation of new technologies and a technological stagnation or even decline in which paradoxically investments in R&D are transformed in losses. In the third one there is an attitude to accept the risk and to make high R&D investments in order to have a probable successful technology because of carrying out of a high number of R&D projects. Large R&D investments probably generate successful technologies encouraging further large investments in R&D. In this last case it is possible that the number of generated technologies will be not limited by availability of capitals, but by human resources and structures in the territory. Such limitation is rare but it existed in the 2nd world war as in the case of the Manhattan Project for nuclear weapons. For example, Battelle Columbus Laboratories participated to this project, studying uranium metallurgy, making available between 1942 – 1946 a staff of 400 researchers working on the base of three shifts per day to exploit fully the available instrumentation for the research (Bohem, Groner1972).

Generation of new technologies

The success of R&D projects in generating new technologies appears stochastic and even largely independent on the cost of development of a new technology. That makes difficult to evaluate a rate of formation of new technologies in respect to the number of financed R&D proposals *r* that could vary sensibly as a function of time and considered territory. In a certain way the generating rate of new technologies may be favored by an efficient selection and financing discussed previously. On the other side it may depend on strategies and management of R&D projects. Best practices in R&D projects management and selection have been developed in the thirties of the past century in large contract research laboratories such that of Battelle (Boehm, Groner 1972), based on an entrepreneurial view of research, and a matrix organization of work consisting in a vertical management of competences and lateral management of projects using the various necessary competences, organization now observed largely in firms and universities of the Silicon Valley (Bonomi 2016).

Generation of successful technologies

The rate of generation of successful technologies *v* is the only parameter that enjoys some indications from statistical studies. These studies show the existence of skew distributed outcomes evaluating various sets of patents (Scherer, Haroff 2000). As in the case of new technologies, appearance of successful technologies is not linked to the amount of investments made for their development showing also a stochastic behavior. Successful technologies may be the result of valid market studies, however, uncertainness of markets estimations is the most hard factor to be reduced during R&D activity until the final phase of industrialization and use of the new technology (Scherer 1999). Good financing strategies are also influencing the success of new technologies as we have previously discussed.

In conclusion, considering as objective the generation of new successful technologies and then an economic growth, the model suggests that the most influencing parameter is the generation of combinatory possibilities I_T coming from previous R&S activity (GRDK) and external information combined by an efficient innovation system characterized by a high value of ISE. A higher number of combinatory possibilities may be obtained also by reducing the fading effect by conserving past GRDK and scientific and technical information for their exploitation. The reaching of the objective, however, depends also by existence at the same time of a suitable strategy of financing and availability of investment capitals and, in a certain measure, also by a good management of R&D.

6 CONCLUSION

The mathematical model of R&D, despite being a rough simulation of the R&D activity, offers some interesting results concerning the effects of R&D investments and its possible application for the development of a territory. A first result concerns the necessity of exceeding a critical number of projects, equivalent to a critical level of R&D investments, in order to trigger a real development of a territory. These levels of critical investments are far easily definable quantitatively, and depend on various aspects of the territory that may include type of technological sectors of its industry and attitude to an entrepreneurial view of research. Another important factor for improvement of growth with R&D concerns maximizing of exploitation of GRDK and external information by the presence in a territory of laboratories for R&D, industrially independent, but offering R&D projects or start up to industry through spin off from these laboratories. Finally, the mathematical model shows clearly the high complexity of the relation between R&D investments and growth characterized by an exponential growth function, but presenting a discontinuity at the low values of investments, characterized by absence or by a weak growth. Such view is quite different from the continuous dependence between R&D investments and growth assumed often in econometric models describing this relation.

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8 FIGURES

Figure 1. Schematic view of the technological model of the R&D process

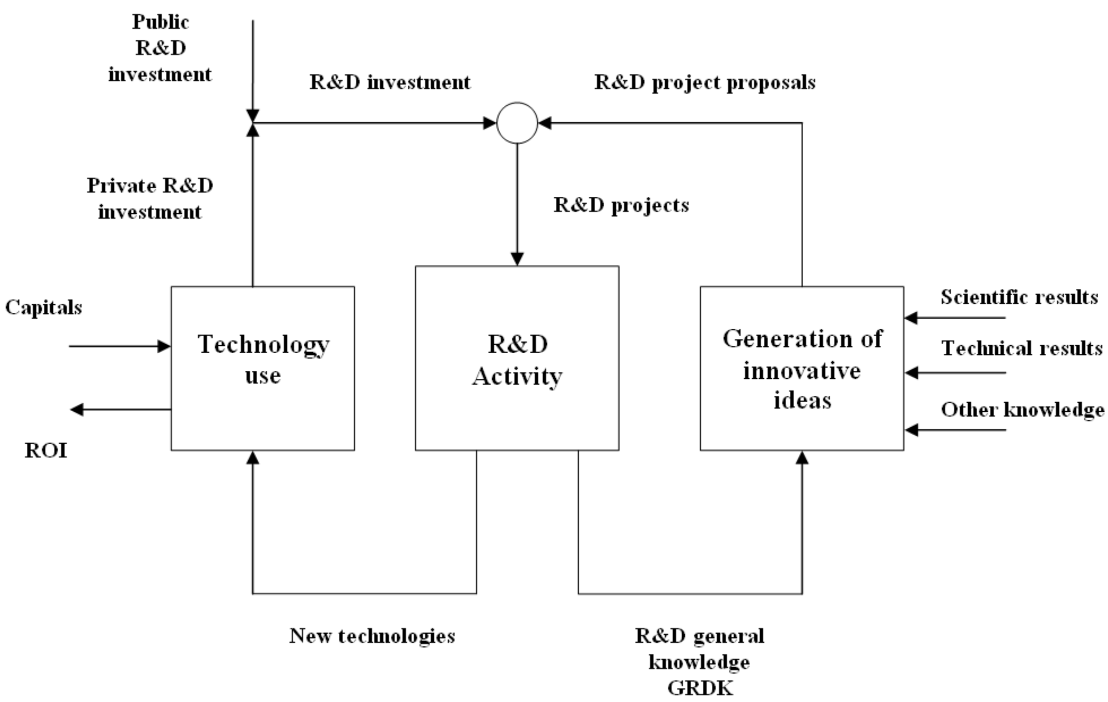
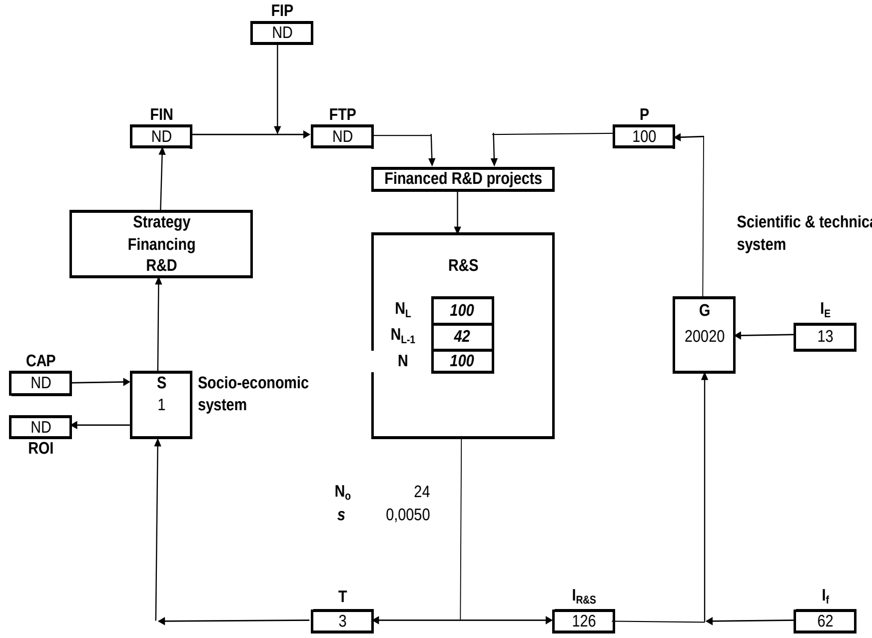


Figure 2. View of the mathematical model of R&D represented in an Excel® sheet



Parameters not determined by the Model (ND)		Parameters of the model		Results of the model	
ROI	Return of Investment	$I_{R\&S}$	Information packages from R&D projects	T	number of new technologies
CAP	Industrialization capital	I_f	Information packages from previous cycles	S	number of successful technologies
FIN	Private R&D investment	I_E	External information packages	N_L	number of projects of present cycle
FIP	Public R&D investment	G	Total number of information packages	N_{L-1}	number of projects of previous cycle
FTP	Total R&D investment	P	Total number of R&D proposals	N	number of projects

Figure 3. Minimum number of initial R&D projects necessary to obtain a number of R&D projects higher than the initial one in the first cycle of activity

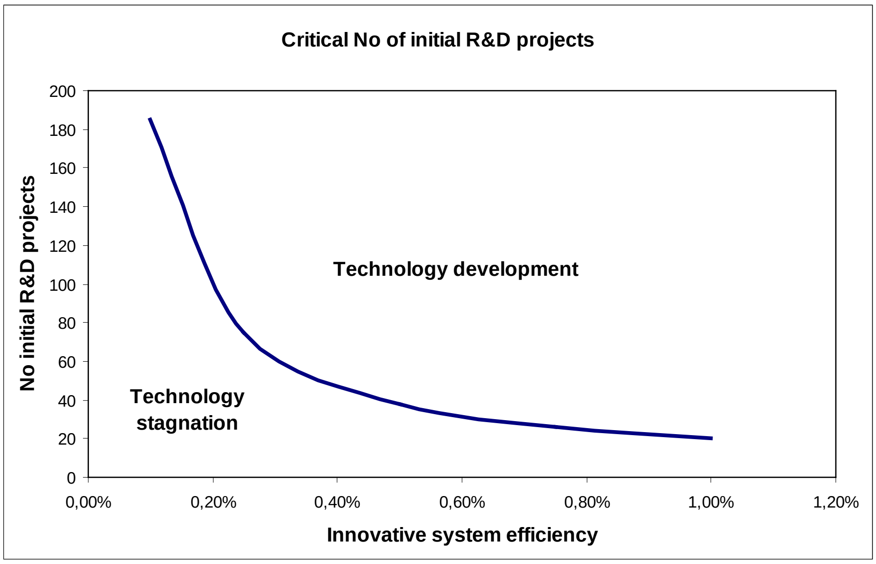


Figure 4. Minimum number of initial R&D projects necessary to obtain at least one successful technology within five cycles of activity

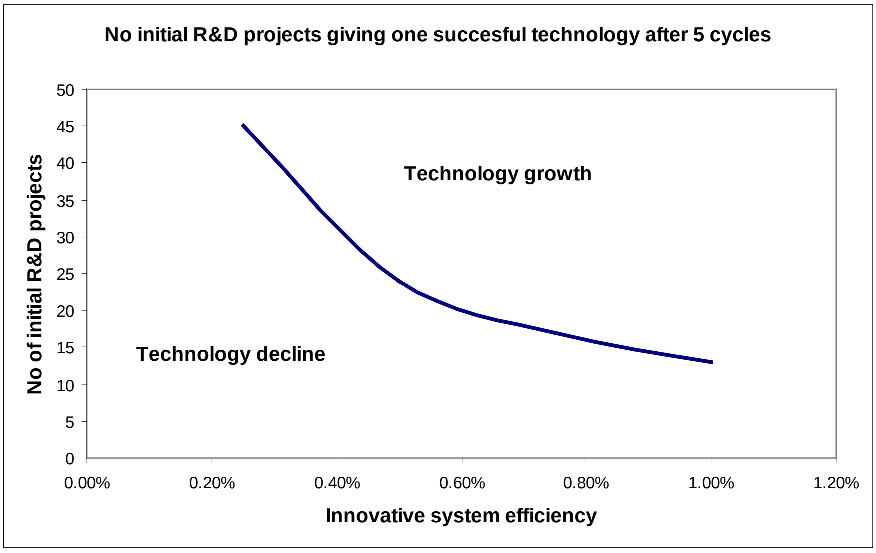


Figure 5. Minimum number of R&D projects necessary to produce at least one new or successful technology within five cycles of activity

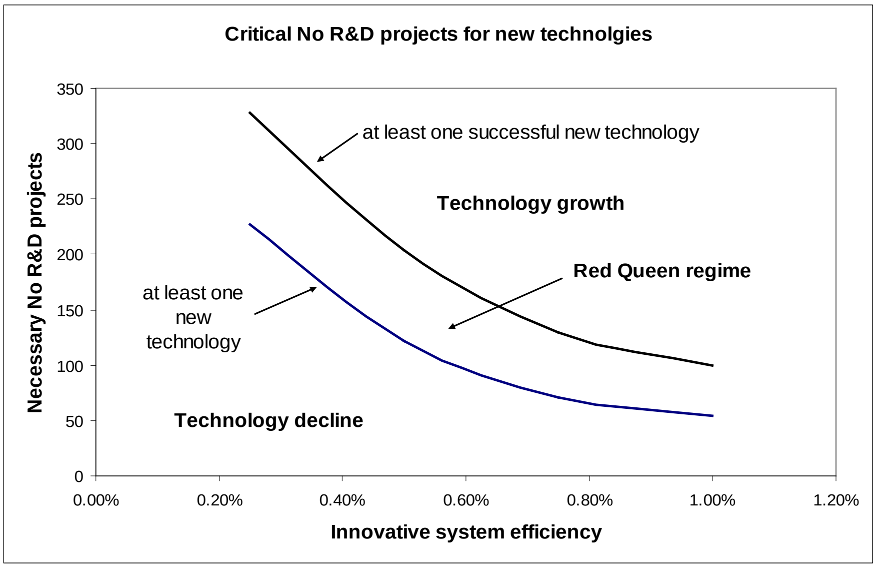


Figure 6. Number of new and successful technologies as a function of number of successive cycles and for different values of ISE (parameter s)